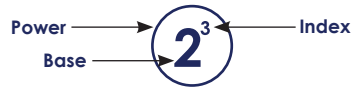
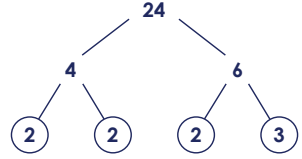
KPI 8.01 Powers and Roots

1) Square number	The result of multiplying a number by itself. It will always be positive. The first 12 square numbers are: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121,144.	**2) Square root**	The opposite of squaring a number to find the original factor. E.g. $\sqrt{64} = 8$ or -8 because $8^2 = 64$ and $(-8)^2 = 64$
3) Cube number	The result of multiplying a number by itself, then itself again. The first 10 cube numbers are: 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000.	**4) Cube root**	The opposite of cubing a number to find the original factor. E.g. $\sqrt[3]{8} = 2$ because $2^3 = 8$ Note: $(-2)^3 = -8$ so $\sqrt[3]{8} \neq -2$
5) Index notation	**Example** $a \times a \times a \times a = a^4$. The number 4 is called the index (plural indices). This tells us how many times the "base" a has been multiplied by itself.		
6) Multiplying powers	$a^m \times a^n = a^{m+n}$ ADD the powers only if the bases are the same. E.g. $a^5 \times a^3 = a^{5+3} = a^8$	**7) Dividing powers**	$a^m \div a^n = a^{m-n}$ SUBTRACT the powers only if the bases are the same. E.g. $a^6 \div a^2 = a^{6-2} = a^4$
8) Indices with brackets	$(a^m)^n = a^{m \times n}$ MULTIPLY the powers. E.g. $(a^3)^5 = a^{3 \times 5} = a^{15}$	**9) Indices with brackets**	$(ab)^n = a^n \times b^n$ Raise each number or variable to the same power. E.g. $(2p)^4 = 2^4 \times p^4 = 16p^4$
10) Power of 0	$a^0 = 1$. Any number or variable to the power of zero equals 1.	**11) Power of $\frac{1}{2}$**	$a^{\frac{1}{2}} = \sqrt{a}$ E.g. $16^{\frac{1}{2}} = \sqrt{16} = 4$

KPI 8.02 Prime Factorisation

1) Prime numbers	A prime number only has two distinct factors: 1 and itself. 2 is the only even prime number. 1 is not a prime number. Prime numbers between 1 and 100: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.		
2) Prime factor decomposition	The process of expressing a number as a product of its prime factors. $24 = 2 \times 2 \times 2 \times 3 \rightarrow 24 = 2^3 \times 3$	**3) Prime factor trees**	

KPI 8.03 Rounding

1) Significant figures

The total number of digits in a number, not counting zeros at the beginning of a number or at the end of a decimal number.
345 000 has 6 significant figures.
0.3047 has 4 significant figures.
10.500 has 3 significant figures.

2) Rounding to significant figures

Round to...	0.007638 to 3 sf	0.007638 to 2 sf	0.007638 to 1 sf	2.0507 to 3 sf	2.0507 to 2 sf	2.0507 to 1 sf
Answer	0.00764	0.0076	0.008	2.05	2.1	2

3) Estimate

Find a rough or approximate answer by calculating with numbers rounded to one significant figure.
e.g. 2.3 x 18.4 ≈ 2 x 20 = 40
≈ "approximately equal to"

KPI 8.04 Fractions

1) Converting an improper fraction to a mixed number

$$\frac{15}{7} = 2\frac{1}{7}$$

2) Converting a mixed number to an improper fraction

$$3\frac{4}{5} = \frac{(3 \times 5) + 4}{5} = \frac{19}{5}$$

3) Adding and subtracting fractions

Make the denominators the same (find the LCM).
Use equivalent fractions to ensure fractions have a common denominator.
Add/subtract the numerators only.

$$\frac{2}{7} + \frac{2}{5} = \frac{10}{35} + \frac{14}{35} = \frac{24}{35}$$

4) Multiplying fractions

Multiply the numerators.
Multiply the denominators.
Simplify where possible.

$$\frac{4}{5} \times \frac{3}{8} = \frac{12}{40} = \frac{3}{10}$$

5) Dividing fractions

Keep the first fraction the same.
Change the second to its reciprocal.
Multiply the fractions.
Simplify or convert to a mixed number where possible.

$$\frac{4}{5} \div \frac{3}{8} = \frac{4}{5} \times \frac{8}{3} = \frac{32}{15} = 2\frac{2}{15}$$

KPI 8.05 Solving Equations 1

1) Inverse operations	Addition and Subtraction are inverse operations. Multiplication and Division are inverse operations. Squaring and taking the square root are inverse operations.	**2) Variable**	A letter used to represent any number.
3) Coefficient	The number to the left of the variable. This is the value that we multiply the variable by. $4x$ ➔ The coefficient of x is 4. x ➔ The coefficient of x is 1.	**4) Term**	A single number, variable or numbers and variables multiplied together.
5) Collecting like terms	Combining the like terms in an expression. $7x + 3y - 2x$ is simplified to $5x + 3y$.	**6) Expression**	A mathematical statement which contains one or more terms combined with addition and/or subtraction signs E.g. $4x + 3y$.
7) Linear equation	Contains an equals sign (=) and has one unknown. E.g. $5x - 2 = 2x + 7$.		

8) Solve

Use inverse operations to find the solution of an equation.

E.g. 1. (One step)

$\frac{x}{4} = 12$

x4 x4

$x = 48$

E.g. 2. (Two step)

$3p - 7 = 8$

+7 +7

$3p = 15$

÷3 ÷3

$p = 5$

E.g. 3. (Unknown on both sides)

$2x + 10 = 19 - 9x$

+9x +9x

$11x + 10 = 19$

-10 -10

$11x = 9$

÷11 ÷11

$x = \frac{9}{11}$

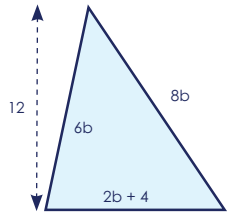
9) Form and solve a linear equation

E.g. 1
Jake is y years old. Lilly is 15. Kobe is 3 years younger than Jake. They have a total age of 36. Work out their individual ages.

$y + 15 + y - 3 = 36$
$2y + 12 = 36$
$2y = 24$
$y = 12$

Jake: 12, Lily: 15, Kobe: 9

E.g. 2
The area of the triangle is 120 cm². Find the value of b.



$\frac{12(2b + 4)}{2} = 120$

$\frac{24b + 48}{2} = 120$

$12b + 24 = 120$

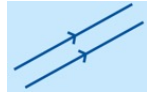
$12b = 96$

$b = 8\text{cm}$

KPI 8.06 Angles in Parallel Lines 1

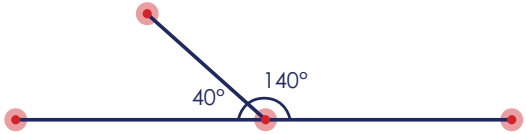
1) Parallel lines

Always equidistant.
Parallel lines have the same gradient.
They never meet however far they are extended.



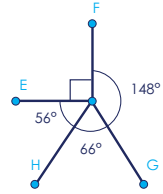
2) Angles on a straight line

Angles on a straight line sum to 180°



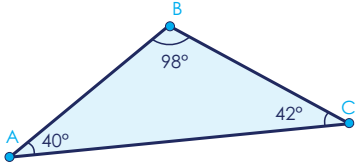
3) Angles around a point

Angles around a point sum to 360°



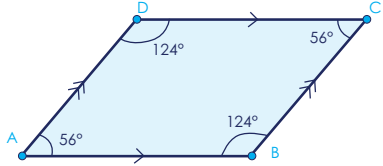
4) Angles in a triangle

Angles in a triangle sum to 180°



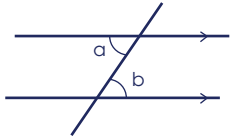
5) Angles in a quadrilateral

Angles in a quadrilateral sum to 360°



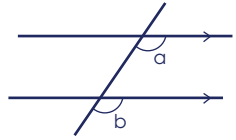
6) Alternate angles

Alternate angles are equal, so a = b



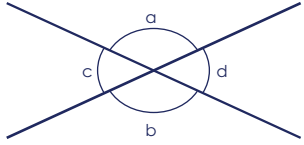
7) Corresponding angles

Corresponding angles are equal, so a = b



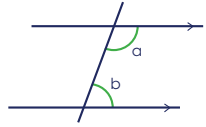
8) Vertically opposite angles

Vertically opposite angles are equal, so, a = b and c = d



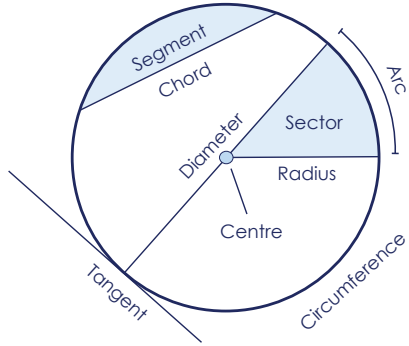
9) Co-interior angles

Co-interior angles sum to 180°, so a + b = 180°



KPI 8.07 Circumference

1) Diameter	A straight line going straight through the centre of the circle and touching the circumference at each end.
2) Radius **Plural: radii**	A straight line joining the centre to the circumference.
3) Chord	A straight line joining any two parts of the circumference.
4) Tangent	A straight line that touches the circumference at a single point.
5) Arc	A section of the circumference.
6) Sector	The area bound by two radii and an arc.
7) Segment	The area bound by the circumference and a chord.



8) Circumference

The perimeter of the circle.
C = π ×diameter
C = π d

d = 5cm

c = πd
c = π x 5
c = 5π cm

c = 15.70796327cm
c = 15.7cm (3*sf*)

9) π (Pi)

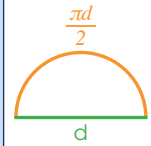
The ratio of a circle's circumference to its diameter.

It has an estimated value of $\frac{22}{7}$ or 3.14 rounded to 3 significant figures.

10) Revolution

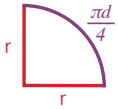
A revolution is a full turn of a circle.
The distance covered by one revolution is equal to the circumference of the circle.

13) Semi circle



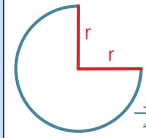
Perimeter $\frac{\pi d}{2}$ + d

12) Quarter- circle



Perimeter $\frac{\pi d}{4}$ + 2r

14) Three-quarter circle



Perimeter $\frac{3}{4}\pi d$ + 2r

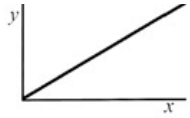
KPI 8.08 Direct Proportion

1) Proportion

A relationship between two quantities.

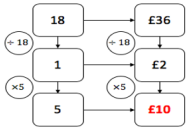
2) Direct proportion

A relationship between two variables where, as one increases, the other also increases.
The graphical representation of this relationship is a straight line through the origin.



3) Unitary method

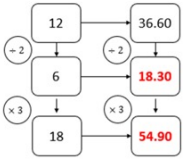
To find the value of one unit first.



5) Best buy

Better value for money means that the cost is cheaper when buying an identical item or amount. Equal quantities must be compared.

4) Multiple intersections



6) Recipes

Option 1: Find the amount of ingredients needed for a specific number of people.
Option 2: Find how much of the recipe can be made with the quantities available in the question.

KPI 8.09 Fractions, Decimals and Percentages

1) Common conversions

Fraction	Decimal	Percentage
$\frac{1}{10}$	0.1	10%
$\frac{1}{8}$	0.125	12.5%
$\frac{1}{5}$	0.2	20%
$\frac{1}{4}$	0.25	25%
$\frac{1}{3}$	0.33333....	33.3% (1dp)
$\frac{1}{2}$	0.5	50%
$\frac{3}{4}$	0.75	75%
$\frac{1}{1}$	1	100%

2) Fraction to decimal

Divide the numerator by the denominator.

$\frac{1}{5} \rightarrow 1 \div 5 \rightarrow 5\overline{)1.0}\ 0.2$

3) Decimal to percentage

Multiply by 100 and add the percentage symbol.

$0.09 \rightarrow 0.09 \times 100 = 9\%$

4) Percentage to fraction

Write the percentage as the numerator and make 100 the denominator. Simplify if possible.

$30\% \rightarrow \frac{30}{100} = \frac{3}{10}$

4) Percentage change

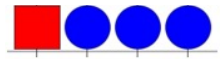
Percentage Increase or Decrease =

$\frac{\text{Change}}{\text{Original}} \times 100$

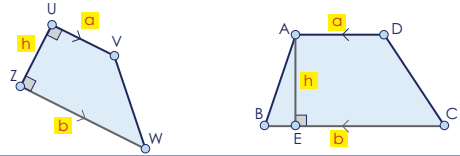
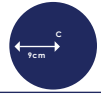
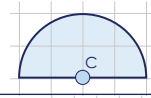
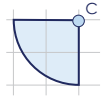
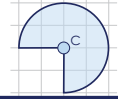
KPI 8.10 Percentages Calculations

1) Multiplier	A percentage written as a decimal is the percentage multiplier.	**2) Percentage of an amount with a calculator**	The percentage multiplier multiplied by the amount.
3) Percentage change	$\frac{\text{difference}}{\text{original}}$ x 100	**4) Reverse percentages**	$\text{original} = \frac{\text{new amount}}{\text{multiplier}}$

KPI 8.11 Ratio 1

1) Ratio	A part-to-part comparison. The ratio of a to b is written a:b	**2) Ratio as a fraction**	Fraction of shapes which are squares: $\frac{1}{4}$  Fraction of shapes which are circles: $\frac{3}{4}$
3) Equivalent ratios	Found by multiplying or dividing all parts of the ratio by the same number.		
4) Simplifying ratios	Ratios can be simplified by dividing each part of the ratio by the same number. ÷5 ↶ 25 : 15 / 5 : 3 ↷ ÷5	**5) Sharing into a given ratio**	Add the parts together. Divide the total by this. Multiply this by each part of the ratio. Share £18 in the ratio of 5:4 Add the part → 4 + 5 = 9 parts £18 ÷ 9 = £2 → 1 part = £2 5 parts: 5 x £2 = £10 4 parts: 4 x £2 = £8 £10: £8
6) Unitary Ratio	Write the ratio 5:3 in the form 1:n ÷5 ↶ 5 : 3 / 1 : $\frac{3}{5}$ ↷ ÷5		

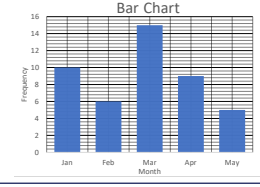
KPI 8.12 Area of Circles

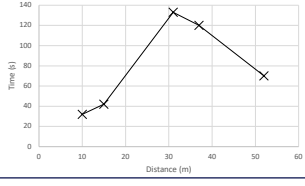
1) Trapezium	Quadrilateral with one pair of parallel sides.	**2) Isosceles trapezium**	Quadrilateral with one pair of parallel side and two right angles.
3) Area of trapezium	Sum of the parallel sides. Divide by 2. Multiply by the vertical height.	$A = \left(\frac{a+b}{2}\right) \times h$	
4) Area of a circle	$A = \pi r^2$ $A = \pi \times 9^2$ $A = 81\pi \text{ cm}^2$ 	**5) Area of a semi-circle**	$A = \frac{\pi r^2}{2}$ 
6) Area of a quarter-circle	$A = \frac{\pi r^2}{4}$ 	**7) Area of a three-quarter circle**	$A = \frac{3\pi r^2}{4}$ 

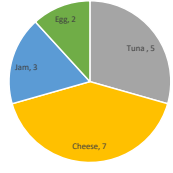
KPI 8.13 Statistics 1

1) Frequency table	A table showing how often (frequent) something occurs. Can include tally charts.	**2) Bar chart**	A way of displaying data, using horizontal or vertical bars which are the same width and have gaps between them. Data can also be presented in dual and composite bar charts in which case a key word would be used.
3) Line graph	Uses lines to join points on a graph to represent a data set.	**4) Pie chart**	Method of displaying proportional information by dividing a circle up into different-sized sectors.
5) Stem and Leaf diagrams	Presents data in a table where the place value columns are split. For example, the tens and the ones columns may be split where the tens become the "stem" and the ones become the "leaf". Stem and lead diagrams come with a key and must always be written in order.		

Score	Tally	Frequency (*f*)
1	||||	4
2	𝍸 ||||	9
3	𝍸 |	6
4	𝍸 |||	8
5	|||	3
6	|	1







12	5
34	31
27	22
19	6
39	40

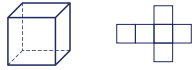
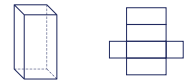
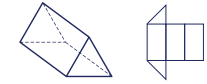
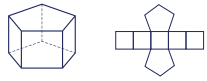
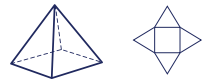
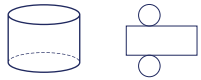
0	5 6
1	2 9
2	2 7
3	1 4 9
4	0

Key
2 | 9 = 29

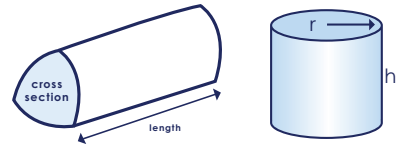
KPI 8.14 Averages and spread

1) Average	The central or typical value in a data set. There are three types of averages: mode, median and mean.	**2) Mode**	The most common/frequent value from a set of data. Mode of 3, 3, 6, **7, 7, 7**, 8, 9, 10 = 7
3) Median	The middle value when the data is in order. Median of 9, 5, 15, 6, 8 → 5, 6, **8**, 9, 15 = 8	**4) Mean**	Add up all the numbers and divide the total by how many numbers there are. Mean of 7, 8, 9: $\frac{7+8+9}{3} = \frac{24}{3} = 8$
5) Range	A measure of the spread of the data, = *Largest Value − Smallest Value*.		
6) Reversing the mean	If we have the mean but one of the data points is missing, we can find the missing value by: 1) Multiplying the 'mean' by the number of data points to get the total of the values; 2) Subtracting the sum of the known values from the total of all values.	E.g. The mean of three numbers is 5. Two of the numbers are 3 and 10. Find the third value. Total of the values: 5 x 3 = 15 15 – (3 + 10) = 2 The third value is 2	

KPI 8.15 3D Visualisation

1) Face	A face is a single flat surface.	**2) Edge**	An edge is a line segment between faces.	**3) Vertex**	A vertex is a corner.
4) Cube	6 faces 12 edges 8 vertices 	**5) Cuboid**	6 faces 12 edges 8 vertices 	**6) Triangular prism**	5 faces 9 edges 6 vertices 
7) Pentagonal prism	7 faces 15 edges 10 vertices 	**8) Square-based pyramid**	5 faces 8 edges 5 vertices 	**9) Triangular -based pyramid**	4 faces 6 edges 4 vertices 
10) Cylinder	3 faces 2 edges 0 vertices 	**11) Cone**	2 faces 1 edge 1 vertex 	**12) Sphere**	1 face 0 edges 0 vertices 

KPI 8.16 Volume

1) Volume	The volume of a solid body is the amount of 'space' it occupies. It is measured in cubic units e.g. cubic centimetres (cm^3).		
2) Volume of a prism	Volume of a prism = area of cross section × length Volume of cylinder = $\pi r^2 h$ 	**3) Units of capacity**	1 L = 1000 ml; 1 L = 1000 cm^3